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DA-003-001617

B. Sc. (Sem. VI) (CBCS) Examination

April / May – 2015

Mathematics - Paper - 602(A)

(Analysis-2 & Abstract Algebra-2)

Faculty Code : 003
Subject Code : 001617

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Answer all M.C.Q. in answer book.
(2) Figures on the right side indicate marks.

1 Answer all M.C.Q.: **20**

(1) X is Metric space and $E \subset X$. For Fixed $\delta > 0$, collection $\{N(a, \delta) \mid a \in E\}$ is __________ of E.

(A) Closed cover

(B) Not cover

(C) Open cover

(D) None of these

(2) X is Discrete metric space and $E \subset X$. E is compact if E is :

(A) Infinite subset

(B) Finite or Infinite subset does not matter

(C) Finite subset

(D) None of these

(3) Which of the following is connected ?

(A) $R-\{5\}$

(B) $[-5,1)\cup(1,10]$

(C) $\{1,3,5,7.........\}$

(D) None of these

(4) A and B are separated sets of Metric space X then

(A) $A=B$

(B) $A\subset B$

(C) $B\subset A$

(D) $A\cap B=\phi$

(5) Which of the following is totally bounded set ?

(A) Infinite discrete Metric space

(B) Compact Metric Space

(C) Both (A) and (B)

(D) None of these

(6) For $n\in N$, $L(t^n)=$ ____________.

(A) $\frac{n!}{S^{n+1}}$ (B) $\frac{n+1}{S^n}$

(C) $\frac{n!}{S^n}$ (D) $\frac{1}{S^n}$

(7) $L\left[(\sin x-\cos x)^2\right]$ = ____________.

(A) $\frac{S^2-2S+4}{S(S^2+4)}$ (B) $\frac{S^2+2S-4}{(S-1)(S-4)}$

(C) $\frac{2S+4}{S(S^2+4)}$ (D) $\frac{3S+2}{S(S^2+4)}$

(8) $L\left[\sqrt{t}\,e^{2t}\right]$ = __________.

(A) $\dfrac{\sqrt{\pi}}{2(S-2)^{3/2}}$ (B) $\dfrac{\pi}{2(S-2)^{1/2}}$

(C) $\dfrac{\pi}{2(S-2)^{3/2}}$ (D) $\dfrac{\sqrt{\pi}}{2(S+2)}$

(9) $L^{-1}\left[\dfrac{1}{4S+5}\right]$ = __________.

(A) $\dfrac{1}{4}e^{-\frac{5}{4}}$ (B) $\dfrac{1}{4}e^{2}$

(C) $\dfrac{1}{4}e^{3}$ (D) $\dfrac{1}{4}e^{-\frac{1}{4}}$

(10) If $L^{-1}\left\{\overline{f}(S)\right\} = f(t)$ and $L^{-1}\left\{\overline{g}(S)\right\} = g(t)$ then

$L^{-1}\left\{\overline{f}(S)\overline{g}(s)\right\}$ = __________.

(A) $\int_0^t f(u)g(t-u)\,du$ (B) $\int_0^t f(t-u)\,du$

(C) $\int_S^\infty \overline{f}(S)\,dS$ (D) $\int_0^\infty \overline{f}(S)\,dS$

(11) G and G' be two groups and $\phi : G \to G'$ is Homomorphism, e' is identity element of G' then $\phi(x) = e'$ iff

(A) $e \in K_{\phi}$ (B) $x \in K_{\phi}$

(C) $K_{\phi} = \{e\}$ (D) Both (A) and (B)

(12) Which of the following is commutative ring without zero divisors and having no multiplicative identity ?

(A) $(Z, +, X)$ (B) $(10Z, +, X)$

(C) $(Q, +, X)$ (D) $(R, +, X)$

(13) Characteristic of rings (i) $Z_3 + Z_4$ (ii) $Z_6 + Z_{15}$ are __________ and _________ respectively.

(A) 3, 6 (B) 12, 30

(C) 4, 15 (D) 12, 90

(14) Zero divisors of ring $(Z_P, +_P, \cdot_P)$ does not exist if p is

(A) Any integer (B) Prime integer

(C) Not prime integer (D) Positive integer

(15) Following is Example of Finite and infinite integral domain respectively

(A) Z_4, Z (B) Z_3, Z

(C) $Z_5, 2Z$ (D) Z_5, Z_7

(16) Which of the following is reducible polynomial ?

(A) $f(x)=x^3+3x+2$ over $Z_5[X]$

(B) $f(x)=x^3+2x^2+2x+5$ over $Z_7[X]$

(C) $f(x)=x^2+x+1$ over $Z_2[X]$

(D) $f(x)=x^2-2$ over $Q[X]$

(17) Let $f(x)=x^3+2x+1$, $g(x)=x^4+3x^2+2$ are polynomials of $Z[X]$. then $\deg(f(x)\cdot g(x))$ is

(A) 7 (B) 12

(C) 6 (D) Does not exist

(18) Elements of principle ideal generated by 2 in ring $(Z_6, +_6, \cdot_6)$ are

(A) 0, 1, 2 (B) 0, 2, 4

(C) 0, 3, 6 (D) 0, 3, 5

(19) Quaternion set Q is not field because :

(A) Q is not ring

(B) Addition is not commutative in Q

(C) Multiplication is not commutative in Q

(D) $(Q-\{0\}, X)$ is not group

(20) If P is prime Number then ring $(Z_P, +_P, \cdot_P)$ is

(A) Ring with unity

(B) Commutative ring

(C) Finite integral domain

(D) All of these

2 (a) Attempt any three : **6**

(1) Show that every Finite subset of any Metric space is compact

(2) Show that Every singleton set of any metric space is connected

(3) Check whether following sets are connected :

(i) $E_1 = \{1, 3, 5, 7, ...21\}$

(ii) $E_2 = (1, 3)$

(4) Check whether set $\{1, 16, 49, ...\}$ is countable set.

(5) Find : $L\left[e^{-3t}(\cos 4t + 3\sin 4t)\right]$

(6) Find : $L^{-1}\left[\dfrac{3S+4}{S^2+16}\right]$.

(b) Attempt any three : **9**

(1) Using definition of compact set show that (0, 2) is not compact.

(2) Let E be closed subset of R. Show that if E is Lower bounded then $\text{glb}\, E \in E$.

(3) Metric space (X, d) is sequentially compact and A be an infinite bounded subset of X. Prove that A has a limit point.

(4) Find : $L[(\cos at)(\cosh at)]$.

(5) If $L\{f(t)\} = \overline{f}(s)$ then prove that

$$L\left[t^n \cdot f(t)\right] = (-1)^n \frac{d^n}{ds^n}\left[\overline{f}(s)\right], n = 1, 2, 3, ...$$

(6) Find : $L^{-1}\left[\dfrac{1}{S(S-1)}\right]$.

(c) Attempt any two : **10**

(1) Show that compact subset of Metric Space is closed and bounded.

(2) $f: R \to R$ where for $x \in R, f(x) = x^2$. Show that f is continuous on R but not uniform continuous on R.

(3) Show that set R of real numbers is uncountable.

(4) Find the Laplace transforms of :

$$f(t) = (t-1)^3, \ t > 1$$
$$= 0 \quad , \ 0 < t < 1$$

(5) Using convolution theorem find :

$$L^{-1}\left[\frac{S}{(S+1)(S^2+1)}\right]$$

3 (a) Attempt any three : **6**

(1) Check whether function $\phi: R \to G$ is Homomorphism, where $(R,+)$ and (G, X) are groups and $G = \{Z \in C \,|\, |Z| = 1\}$.

(2) $(G,*)$ and (G', Δ) be two groups. $\phi: G \to G'$ is Homomorphism. Show that if e is identity element of G then $\phi(e)$ is identity element of G'.

(3) Define characteristic of ring. Write all zero divisors of Z_{34}.

(4) Show that every Field is an integral domain.

(5) $f(x), g(x) \in Z_5[X]$. Where $f(x) = 2x^3 + 4x^2 + 3x + 2$ and $g(x) = 3x^4 + 2x + 4$. Find $f(x) + g(x)$ and $f(x) \cdot g(x)$.

(6) In Quaternion set, Find $\left[(3+2i)(4j+5k)^{-1}\right]$.

(b) Attempt any three : **9**

(1) Let $(G,*)$ and (G', Δ) be two groups $\phi: G \to G'$ is Homomorphism. Prove that if K is Normal subgroup of $\phi(G)$ then $\phi^{-1}(K)$ is Normal sub group of G.

(2) Prove that kernal of Homomorphism $f: G \to G'$ is Normal subgroup of G. where G and G' are groups.

(3) n is Fixed Positive integer m is non-zero element of ring $(Z_n, +_n, \cdot_n)$. Prove that integers m and n are not relatively prime iff m is zero divisor of ring Z_n.

(4) U is non empty set. For $A, B \in P(U)$ $A \cdot B = A \cap B$ and $A + B = (A - B) \cup (B - A)$. Show that $(P(U), +, \cdot)$ is ring and Find characteristics of this ring.

(5) In $Z_5[X]$, divide $f(x) = x^4 + 3x^3 + 2x + 4$ by $g(x) = x - 1$ and Find quotient $q(x)$ and remainder $r(x)$ and Express $f(x)$ as a $q(x)g(x) + r(x)$.

(6) Find irreducible Factors of $4x^2 - 4x + 8$ over $Q[X]$.

(c) Attempt any two : **10**

(1) Show that I is a subring of ring $(M_2(R), +, X)$ but not left or right ideals of ring $(M_2(R), +, X)$ where

$$I = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \Big/ a, b \in R \right\}$$

(2) G_n is set of all non-zero elements of Z_n, which does not contain zero divisors. Show that (G_n, X_n) is a group.

(3) State and prove division algorithm theorem for polynomials.

(4) State and prove Factor theorem and remainder theorem.

(5) Show that $x^3 + 3x^2 - 8$ is irreducible over $Q[X]$.